Atmospherical wavefront phases using the plenoptic sensor (real data).

L.F. Rodríguez-Ramos\textsuperscript{b}, I. Montilla\textsuperscript{b}, J.P. Lüke\textsuperscript{a}, R. López \textsuperscript{b}, J.G. Marichal-Hernández\textsuperscript{a}, J. Trujillo-Sevilla\textsuperscript{a}, B. Femenía\textsuperscript{b}, M. López\textsuperscript{a}, J.J. Fernández-Valdivia\textsuperscript{a}, M. Puga\textsuperscript{a}, F. Rosa\textsuperscript{a}, J.M. Rodríguez-Ramos*\textsuperscript{a}

\textsuperscript{a}Universidad de La Laguna, Canary Islands, Spain.
\textsuperscript{b}Instituto de Astrofísica de Canarias, Canary Islands, Spain.

ABSTRACT

Plenoptic cameras have been developed the last years as a passive method for 3d scanning, allowing focal stack capture from a single shot. But data recorded by this kind of sensors can also be used to extract the wavefront phases associated to the atmospheric turbulence in an astronomical observation.

The terrestrial atmosphere degrades the telescope images due to the diffraction index changes associated to the turbulence. Na artificial Laser Guide Stars (Na-LGS, 90km high) must be used to obtain the reference wavefront phase and the Optical Transfer Function of the system, but they are affected by defocus because of the finite distance to the telescope.

Using the telescope as a plenoptic camera allows us to correct the defocus and to recover the wavefront phase tomographically, taking advantage of the two principal characteristics of the plenoptic sensors at the same time: 3D scanning and wavefront sensing. Then, the plenoptic sensors can be studied and used as an alternative wavefront sensor for Adaptive Optics, particularly relevant when Extremely Large Telescopes projects are being undertaken.

In this paper, we will present the first observational wavefront phases extracted from real astronomical observations, using punctual and extended objects, and we show that the restored wavefronts match the Kolmogorov atmospheric turbulence.

Keywords: CAFADIS, plenoptic, wavefront, lightfield, adaptive optics, 3D, GPU, FPGA

1. INTRODUCTION

Figure 1 shows the classic scheme of a plenoptic sensor: a microlens array samples the focal plane of the lens and makes the image over the sensor. In fact, the plenoptic sensor captures the lightfield of the scene, ie both the spatial distribution as the angle of light rays in the scene. Until now, the most common treatment for plenoptic images consists of applying multistereo computer vision techniques, considering that there are as many points of views as there are pixels behind each microlens (Figure 2)

![Diagram of plenoptic sensor](image)

Figure 1. The 4D lightfield, 2D spatial (x,y) and 2D angular (u,v), is captured by a plenoptic sensor.

*jmramos@ull.es; phone +34 922 318 091; fax +34 922 318 228;
Within a plenoptic frame there is enough information to recreate *a posteriori* a volume of images covering a range of focusing distances, what is known as a focal stack. Attending to a measure of focusing through each ray that crosses that volume it is possible to estimate distances to objects. Algorithms have been developed to tackle all of these processes at video acquisition rate and nowadays it is possible to build around these methods a 3D camera, even one suitable to feed a glasses free 3D display (CAFADIS¹). For the sake of speed parallel hardware is usually employed in the form of Graphics Processing Units and Field Programmable Gates Arrays.

Figure 2. Upper panel: plenoptic frame. Medium panel: recomposed plenoptic frame. Bottom panel: detailed recomposed frame.
Plenoptic methods, of course, have their inconveniences. The valid range of refocusing distances is constrained by the number of pixels that contribute to gather angular information. Moreover the specific placement of centres of depth of field of the resulting images is far from being linear, and is best suited for filming on close distances, which incidentally are a hard task when using stereo techniques. If we increment the number of angular pixels there are less remaining pixels to gather "positional" resolution. This sacrifice cannot be fully overcome, but can be diminished by the use of superresolution methods. Also, when the microlenses array is placed close to the sensor, it is hard to avoid a certain tilt between them. This is the reason that explains the calibration stage that is mandatory previous to any further processing.

2. THE PLENOPTIC SENSOR AS DEPTH SENSOR

Some of the most important science goals of any Extremely Large Telescope are high-redshift galaxies and the re-ionization of the early universe. In both cases, high performance wide-field Adaptive Optics is a critical requirement. For this reason Multi-Conjugated AO systems will be built on those telescopes, but they need several Guide Stars for the real-time compensation of the atmospheric distortions over a wide field. There are not enough bright Natural Guide Stars to cover the whole sky. Therefore, Laser Guide Stars are needed to operate such systems. Among them, Sodium LGSs are widely used. They are generated by the back-scattered light from the sodium layer in the mesosphere. Due to the vertical extension of the layer, the LGS is seen as an elongated source from the Wave Front Sensor subapertures located far from the launching point of the laser. This elongation affects the calculation of the centroid in the WFS and the SNR, and therefore it has to be estimated.

1.1 The mesospheric Na layer.

The altitude and linear vertical size of the Na LGS are usually assumed to 90 and 5-10km respectively, adopting the standard mesospheric sodium levels. But the Na layer is strongly affected by global and local effects, either seasonal or very short time scaled. Sporadic bright layers can appear at different heights with an extension down to 500 meters. These variations may change the Na centroid in a range of cents to thousands meters even in a few minutes interval affecting the centroid algorithm performance. Therefore, it is required to measure on real-time its distribution vs. height. This 3D distribution can be measured with a plenoptic sensor.

The plenoptic sensor, also known as the plenoptic camera, was originally created to allow the capture of the Light Field\(^1\), a four-variable volume representation of all rays and their directions, that allows the creation by synthesis of the image as seen from virtually any point of view.

![Diagram](image)

Figure 3. Depending on how the pixels are combined, we retrieve the focused image of the object placed at one distance or another.
As can be seen in Fig. 3, a microlens array with the same f-number than the telescope is placed at its focus, in such a way that many pupil images are obtained at the detector, each of them representing a slightly different point of view. The use of the same f-number guarantees that the size of the image of the pupil is as big as possible, without overlapping with its neighbour, providing thus optimum use of the detector surface. In order to understand the behavior of the plenoptic sensor, it is important to identify the direct relationship existing between every pupil point \((u,v)\) and its corresponding image for each microlens. Every pupil coordinate \((u,v)\) is imaged through each microlens in a way that all rays will arrive to one of the pupil images, depending only on the angle of arrival. This fact clearly indicates that the image can be reconstructed by post-processing the plenoptic data, selecting the value of the corresponding coordinate at every microlens \((x',y',u',v')\) and building an image with all of them. For depth extraction the plenoptic sensor acts as a multiview stereo system; whereas, for wavefront phase extraction, it can be understood as a global sensor containing as extreme cases the Shack-Hartmann sensor and the pyramid sensor (2x2 microlens array at telescope focus). The tomographic capability of the plenoptic sensor was demonstrated by Rodríguez-Ramos et al. and Montilla et al. Due to the finite sampling of the \((u,v)\) plane the object space is also finitely sampled. Depending on how the pixels are combined, we retrieve the focused image of the object placed at one distance or another. There are different algorithms that refocus more or less planes at different distances. This 3D reconstruction makes possible to retrieve the distances at which the objects are. For this reason it is especially adequate to measure the height variations and vertical distribution of the LGS beacons. This novel approach provides real-time information on the Na layer profile that can be introduced in the reconstruction algorithm to solve the problems derived by the spot elongation, giving a more complete alternative to other methods.

1.2 Measuring the Na profile: laboratory results.

The plenoptic image captured by CAFADIS is shown in Fig. 3. Using an array with \(N \times N\) microlenses and a sensor with \(M \times M\) pixels/microlens creates a \(N^2 \times M^2\) pixels image. This image is processed with the plenoptic software programmed for GPUs, allowing reconstruction times of ms. The fastest variations of the Na profile have been found to be \(\sim 1\)Hz. The super resolution algorithm gives \((2p - 2)\) refocused planes, with \(p\) equal to a prime number smaller than half the number of pixels behind each microlens. For our experiment we used a F/19 lens with a diameter of 17.8 mm, a microlens array with 110 micron pitch, and a sensor with 12 pixels behind each microlens. In Fig. 4, we can see the map of distances, with each grey zone corresponding to the part of the image that is focused at each distance. The predicted distances correspond to the distances measured in the lab, within 1 mm accuracy. Extrapolating to the sky, that means a 200 m precision, covering 28 km, in real-time. Ideally we would like to resolve more planes, and it is possible to increase the depth resolution with the Radon algorithm. It would give \((2N - 1)\) refocused planes with \(N\) equal to the number of pixels.

Figure 4. Laboratory results: Left, the image taken with a plenoptic sensor; center, the predicted depths; right, the map of distances, each grey zone gives the depth of a focused plane. As can be seen, the predicted distances correspond to the distances measured in the lab, within 1 mm accuracy.
3. THE PLENOPTIC WAVEFRONT SENSOR.

An alternative procedure, that turns out to be efficient, as well as complementary in fields like Adaptive Optics for Astrophysics, employs plenoptic cameras as wavefront phase sensors. Previous articles have shown, on computer simulations, the plenoptic sensor's ability to recover the wavefront phase at pupil telescope so as tomographically. On this occasion, we present the first plenoptic data acquired in an astrophysical observatory.

3.1 Observational data.

The telescope used was the Optical Ground Station (OGS) of the Observatorio del Teide (Canary Islands). Is a 1 meter aperture telescope and 38.95 m. of focal length at its Coudé focus. The telescope was pointed to the Moon, as extensive object model to perform tomographic analysis of the atmosphere, and take advantage of the full capacity of plenoptic sensor as wavefront phase sensor. Figure 5 shows an image, acquired inside the stability time of the atmosphere, in which a crater can be clearly appreciated. The aperture of the telescope is void, and therefore, the microlens sampling the image at focus corresponds to a sample of pupils annular rather than circular. Even a superposition of these annular pupils clearly show the spider holding the secondary mirror.

![Figure 5. Plenoptic frame from the Moon. A crater can be clearly appreciated.](image)

Figure 6 shows the recomposed plenoptic frame as is done in section 1 of this article. It also shows in detail a piece of the frame recomposed. As the Moon can be considered infinitely far from the telescope, the differences seen between the different sub-images of the same crater can only be due to differential turbulence suffered by the rays that form each subimage. It would be therefore possible to recover the phase from various points of view, using correlations, and consequently to estimate the phase of wavefront tomographically, like a wide field low order Shack-Hartmann (LOWFS).
Figure 6. The recomposed plenoptic frame and a detailed portion of it. The differences between subimages are due to the differential turbulence traversed by the rays that form every subimage.

3.2 Punctual source

Pointing to a point source at infinity (Vega), we can only recover the phase of the wavefront from a point of view, that is, the pupil phase, just as classical sensors do (Shack-Hartmann, pyramid or curvature). Figure 7 shows a plenoptic frame of Vega, in the time of stability of the atmosphere. Figure 8 shows the frame recomposed and in detail. Differences can be seen between the different images, which can only be due to differential turbulence that rays have suffered.

Figure 7. Left: Plenoptic frame of a punctual source: Vega. Right: restored wavefront phase.
3.3 A descriptive approach to restore phases.

The plenoptic wavefront sensor samples the signal, the complex amplitude of the electromagnetic field at the image plane, to obtain the gradient of the pupil wavefront phase map: \( \tilde{S}(u, v) \).

The link [http://www.cafadis.ull.es/demo/vega_pleno_wavefront_gradients.avi](http://www.cafadis.ull.es/demo/vega_pleno_wavefront_gradients.avi) shows a sequence of the estimated phase gradients from Vega. From these gradients estimation, the wavefront phase \( \Phi(u, v) \) can be recovered (Figure 7, right), for example, using an expansion over complex exponential polynomials (Marichal\textsuperscript{6} et al.):

\[
\Phi(u, v) = \sum_{p, q=0}^{N-1} a_{pq} Z_{pq}(u, v) = \sum_{p, q=0}^{N-1} a_{pq} \frac{1}{N} e^{\frac{2\pi i}{N} (pu + qv)} = FFT^{-1}(a_{pq}),
\]

The gradient is written as:
\[
\tilde{S}(u, v) = \tilde{\nabla} \Phi(u, v) = \frac{\partial \Phi}{\partial u} \hat{i} + \frac{\partial \Phi}{\partial v} \hat{j} = \sum_{p, q} a_{pq} \hat{\nabla} Z_{pq}
\]

Making a least squares fit over the F function:
\[
F = \sum_{u, v=1}^{N} \left[ \tilde{S}(u, v) - \sum_{p, q} a_{pq} \left( \frac{\partial Z_{pq}}{\partial u} \hat{i} + \frac{\partial Z_{pq}}{\partial v} \hat{j} \right) \right]^2,
\]

where \( \tilde{S} \) are experimental data. Then, the coefficients \( a_{pq} \) of the complex exponential expansion in a modal Fourier wavefront phase reconstructor, can be written as:
\[
a_{pq} = \frac{i \cdot p \cdot FFT[S^x(u, v)] + i \cdot q \cdot FFT[S^y(u, v)]}{p^2 + q^2}
\]
And finally the phase can be recovered from the gradient data by transforming backward those coefficients:

$$\phi(u, v) = \text{FFT}^{-1}(a_{pq})$$

So a filter composed by three Fourier transforms must to be done to recover the phase. Because of the telescope annular mask, the Fourier transform introduces big spatial frequencies that produce wrong recovered phases. This can be avoided using Gerchberg iterations.

### 4. RESULTS

In order to demonstrate that the recovered phases correspond atmospheric phases, we have calculated the structure function $D_\phi(\hat{\rho})$ on strings of several thousand phase maps restored:

$$D_\phi(\hat{\rho}) = \langle |\phi(\vec{r} + \hat{\rho}) - \phi(\vec{r})|^2 \rangle$$

which should coincide with the known expression of the Kolmogorov turbulence:  

$$D_\phi(\rho) = 6.88 \cdot \left(\frac{\rho}{\rho_0}\right)^{5/3}$$

Where $\rho = |\hat{\rho}|$ is the polar coordinate and $\rho_0$ is the seeing parameter. As Figure 9 shows, the structure function calculated on the phases recovered from actual data matches the expected according to the law of the Kolmogorov turbulence, allowing us to say that we are measuring atmospheric turbulence from plenoptic frames.

![Structure function at several $T_{\text{int}}$](image)

Figure 9. Matching between the Kolmogorov turbulence law and the structure function of the recovered phases (at different integration times). It can be seen that a very short acquisition time (1.7 ms) implies a bad matching, probably because the noise is too high.
5. CONCLUSIONS.

The main conclusion we can draw from this experience is that the plenoptic sensor can measure phases of atmospheric wavefronts, and therefore can be used in development of Adaptive Optics at the same level as the classical wavefront sensors: Shack-Hartmann, Pyramid, curvature...

But also it has two clear advantages: the correction of defocus when using Laser Guide Stars, as it is capable of measuring the vertical distribution of it, and recovery the phase tomographically, possible when working with extended objects (solar observations and observations with several artificial laser guide stars).

Although new developments are required to analyze the true capacity of the plenoptic sensor for Adaptive Optics, the steps taken and the expected benefits open a very optimistic expectation for future implementation.

ACKNOWLEDGMENTS

This work has been partially funded by national R&D Program (Project AYA2009-13075) of the Ministry of Science and Innovation, by the Agencia Canaria de Investigación, Innovación y Sociedad de la Información (ACIISI) and by the European Regional Development Fund (ERDF).

REFERENCES